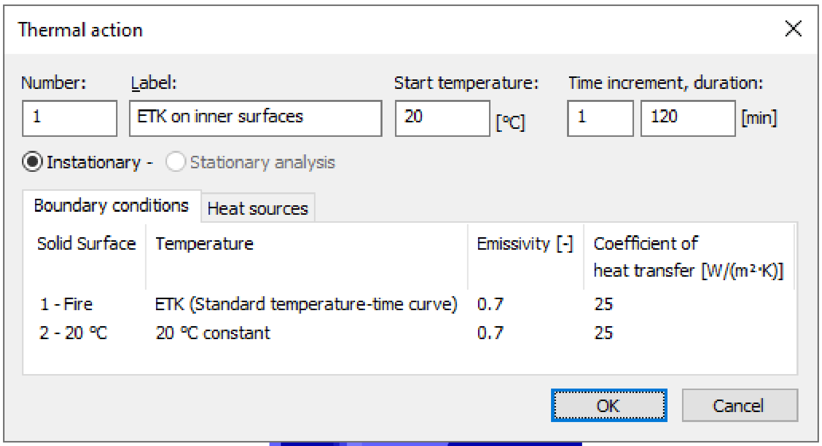
Thermal Analysis
for Solid Structures



Thermal action

Number: 1

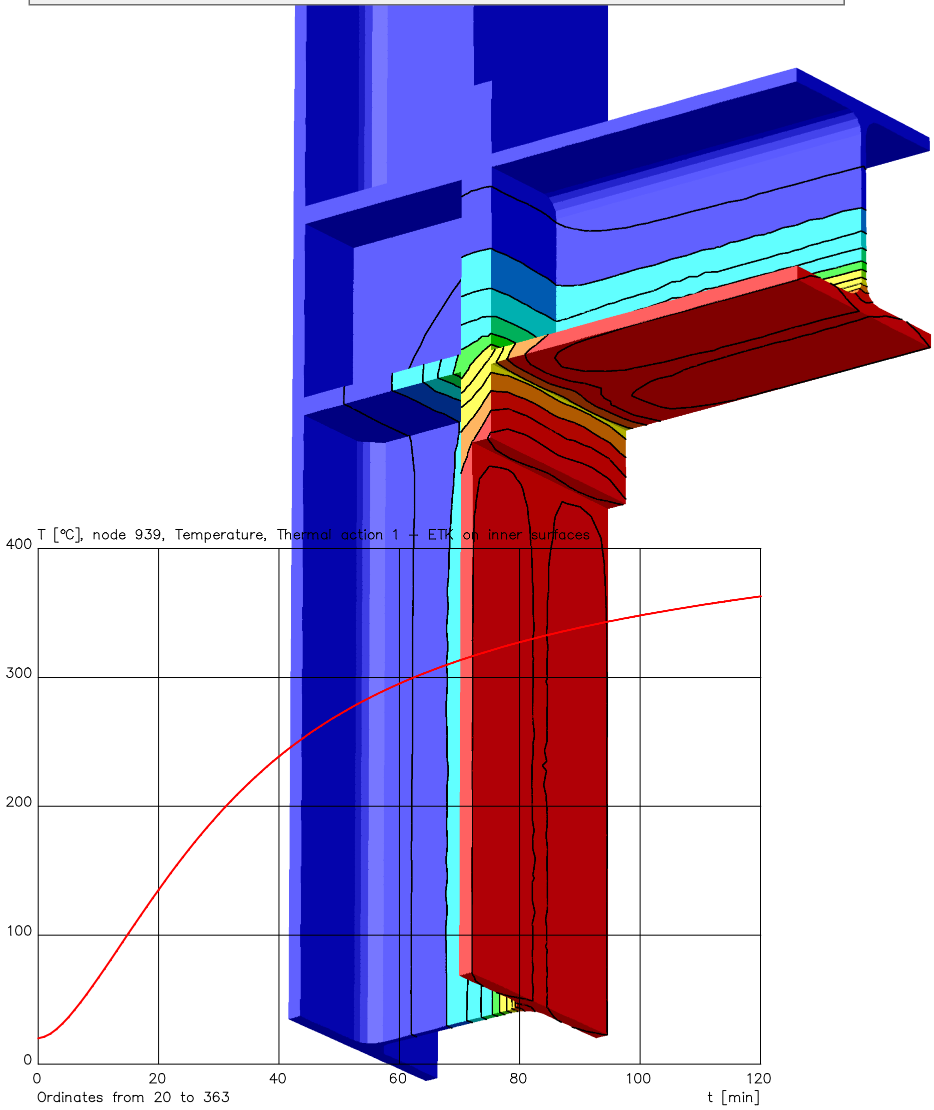
Label: ETK on inner surfaces

Start temperature: 20 [°C]

Time increment, duration: 1 120 [min]

● Instationary - ○ Stationary analysis

Solid Surface	Temperature	Emissivity [-]	Coefficient of heat transfer [W/(m²·K)]
1 - Fire	ETK (Standard temperature-time curve)	0.7	25
2 - 20 °C	20 °C constant	0.7	25



InfoGraph

Software für die Tragwerksplanung

Title image: Temperature distribution at the corner of a frame after 120 minutes of fire exposure on the inner surfaces.

Contents

Thermal Analysis of Solid Structures

Area of Application

The *Thermal analysis* allows the analysis of stationary and instationary temperature distributions in solid structures of tetrahedron elements within the *Finite Elements* program systems. Any three-dimensional geometry can be analyzed.

The material properties are described in the section *Thermal material properties*.

The analysis can be divided up into the following steps:

- The structure geometry is described with model object *Solid*.
- Properties like material and presets for the mesh generation are assigned to each solid model object.
- The thermal material properties are defined in the section dialog.
- After selection of the surface of a solid a surface property can be assigned to it.
- In the dialog *Thermal action* thermal boundary conditions are assigned to the surfaces.
- After the definition of all properties which are relevant for the structure the finite element model is generated with the mesh generator *Tetrahedrons from Solid*.
- Subsequently the thermal analysis is carried out.
- Additionally a static analysis can be performed considering the thermal strains due to a thermal action (load type *Thermal action*).

Multiple independent *Thermal actions* can be defined.

Basics

The calculation of the temperature distribution for transient situations in solid bodies is carried out by solving the thermal conductivity equation as set out by Fourier. This has the following form:

$$\rho \cdot c_{\mathrm{p}} \cdot \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \cdot \left(\lambda \frac{\partial \theta}{\partial x} \right) + \frac{\partial}{\partial y} \cdot \left(\lambda \frac{\partial \theta}{\partial y} \right) + \frac{\partial}{\partial z} \cdot \left(\lambda \frac{\partial \theta}{\partial z} \right) + \dot{q} \tag{1}$$

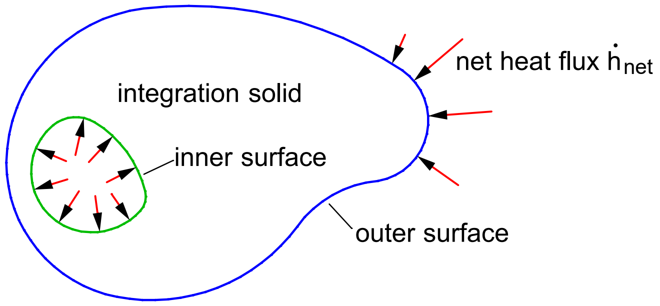
with

t	Time [s]
y, z	Local coordinates [m]

Functions of y, z, t

θ	Temperature [°C]
ρ	Apparent density [kg/m³]
c_{p}	Specific thermal capacity [J/(kg K)]
λ	Thermal conductivity [W/(m K)]
$\dot{q}$	Heat source (power density) [W/m³]

As all factors are dependent on time, the initial value problem needs to be solved in consideration of the boundary conditions. The temperature distribution in the integration solid at the time $t = 0$ is taken as the initial condition.



For the problem being considered here, the thermal actions will be primarily determined by the net heat flux $\dot{h}_{\mathrm{net}}$ [W/m²] normal to the surfaces. This, in turn, comprises a convection and radiation component.

$$\dot{h}_{\mathrm{net}} = \dot{h}_{\mathrm{net,c}} + \dot{h}_{\mathrm{net,r}} \tag{2}$$

The convective component is calculated as follows:

$$\dot{h}_{\mathrm{net,c}} = \alpha_{\mathrm{c}} \left(\Theta_{\mathrm{g}} - \Theta_{\mathrm{m}} \right) \tag{3}$$

with

α_{c}	Coefficient of heat transfer [W/(m² K)]
Θ_{g}	Gas temperature in the region of the component under load [°C]
Θ_{m}	Temperature at the surfaces of the component [°C]

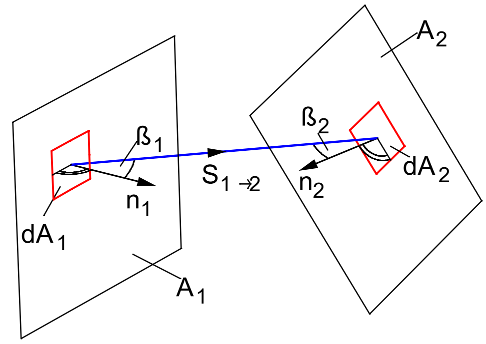
The net heat flux between two surfaces due to radiation is calculated as follows:

$$\dot{h}_{\mathrm{net,r}} = \Phi \cdot \varepsilon_1 \cdot \varepsilon_2 \cdot \sigma \cdot \left[(\Theta_1 + 273)^4 - (\Theta_2 + 273)^4 \right] \tag{4}$$

with

Φ	Configuration factor
ε_1, ε_2	Emissivity of the component surfaces [-]
σ	Stephan-Boltzmann constant [= $5.67 \cdot 10^{-8}$ W/(m^2 K^4)]
Θ_1, Θ_2	Temperature at the component surfaces [°C]

The configuration factor is determined by the geometry of the surface, the distance between the surfaces and shadowing effects. The following figure shows the underlying parameters.



The configuration factor can be determined as follows:

$$\varphi_{1\to2} = \frac{1}{\pi\cdot A_1}\cdot\int_{A_1}\int_{A_2}\frac{\cos\beta_1\cdot\cos\beta_2}{s_{1\to2}^2}\cdot dA_1\cdot dA_2$$

In the case of radiation due to fire follows:

$\Phi = 1$

$\varepsilon_1 = \varepsilon_m$	emissivity of the component surface [-]
$\varepsilon_2 = \varepsilon_f$	emissivity of the flame (here $\varepsilon_f = 1$) [-]
$\Theta_1 = \Theta_r$	effective radiation temperature of the fire (here $\Theta_r = \Theta_g$) [°C]
$\Theta_2 = \Theta_m$	surface temperature at the component boundary [°C]

The gas temperature Θ_g is assumed to be known and is generally defined according to particular temperature-time curves.

The nonlinear boundary problem set by equations (1) to (4) cannot be solved in a closed form. Instead, the finite element method is used to solve the problem. In this case, Galerkin's method followed by partial integration is used to convert the boundary problem described above into a system of variation equations.

The program system uses compatible elements with a fully quadratic approach. This results in a high degree of accuracy, which is largely independent of the mesh geometry.

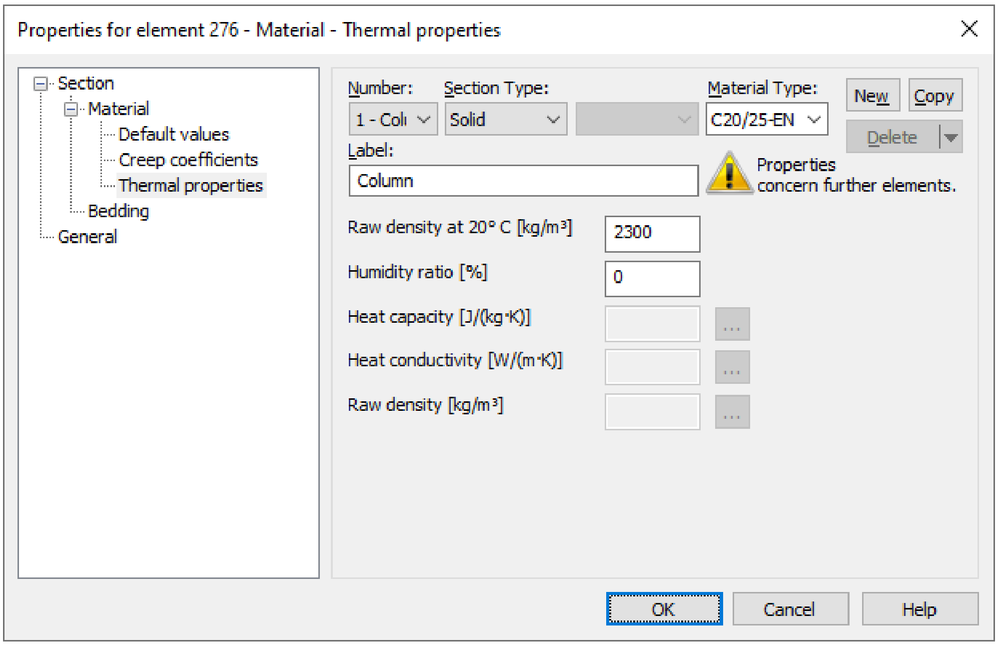
The element approach automatically satisfies the so-called *adiabatic boundary conditions*, meaning that thermal energy is only exchanged where boundary conditions are explicitly specified.

The nonlinear initial value problem is solved using time-step integration in conjunction with an incremental Newton-Raphson algorithm. After successful calculation, the temperature profiles for all times considered are available for further calculation.

Input Data

Thermal Material Properties

The used curves of the temperature dependent material properties are described subsequently. The necessary user-defined values are to be specified in the section dialog.



Properties for element 276 - Material - Thermal properties

Number: 1 - Colı

Section Type: Solid

Material Type: C20/25-EN

Label: Column

Raw density at 20° C [kg/m³]: 2300

Humidity ratio [%]: 0

Heat capacity [J/(kg·K)]

Heat conductivity [W/(m·K)]

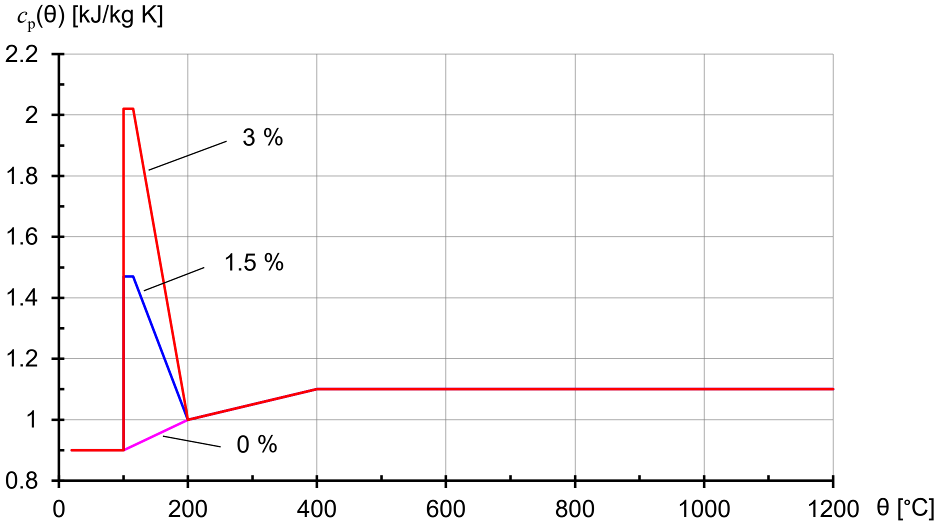
Raw density [kg/m³]

Properties concern further elements.

All material values outside the temperature range between 20 °C and 1200 °C are assumed to be constant.

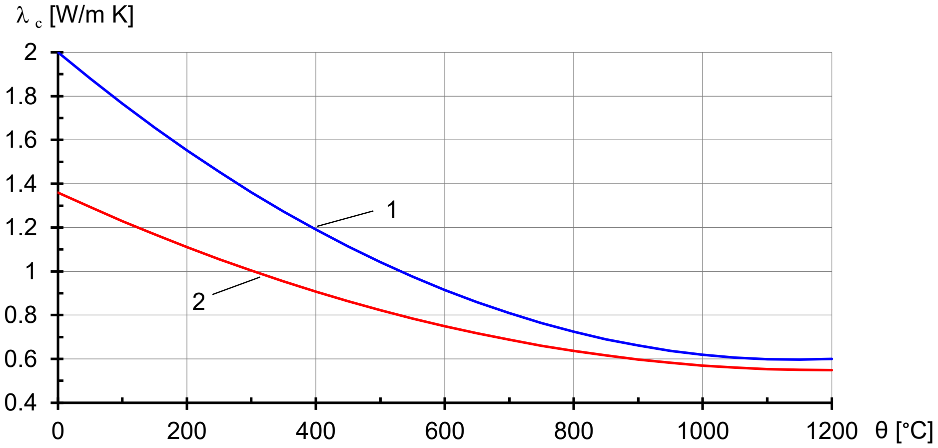
Concrete

All concrete types are assigned the temperature-dependent *specific heat capacity*, the *thermal conductivity* and the *raw density* as specified in EN 1992-1-2. According to DIN EN 1992-1-2/NA:2010-12 the upper limit function of the *thermal conductivity* in 3.3.3 (2) is used for DIN concrete. The lower boundary value is used for other kinds of concrete. The *humidity ratio* and *raw density at 20° C* can be specified by the user.

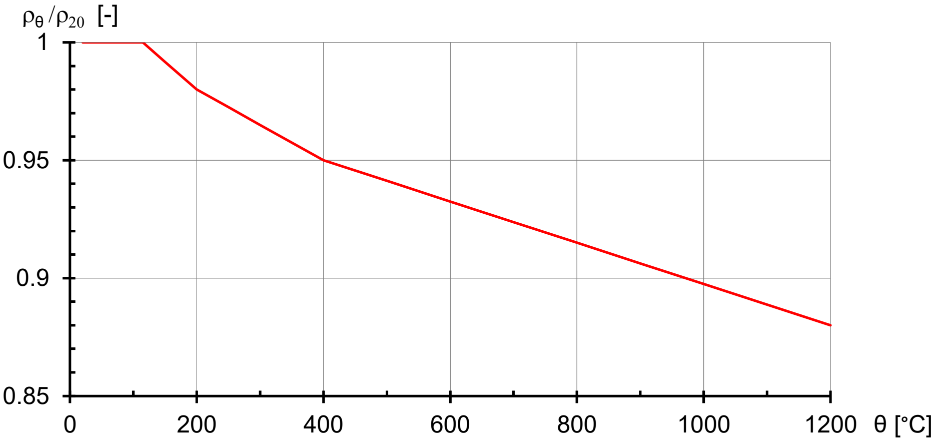


The specific heat capacity of siliceous and calcareous concrete $c_p(\theta)$ depending on the temperature (moisture content u = 0, 1.5 and 3 % by weight)

Moisture contents between the values specified will be linearly interpolated.



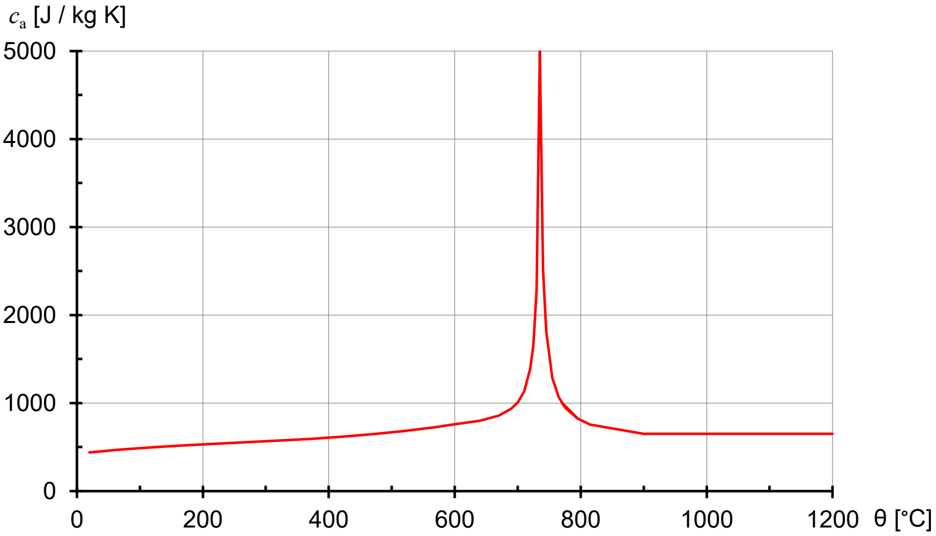
Temperature-dependent thermal conductivity
1: upper limit (DIN concrete); 2: lower limit



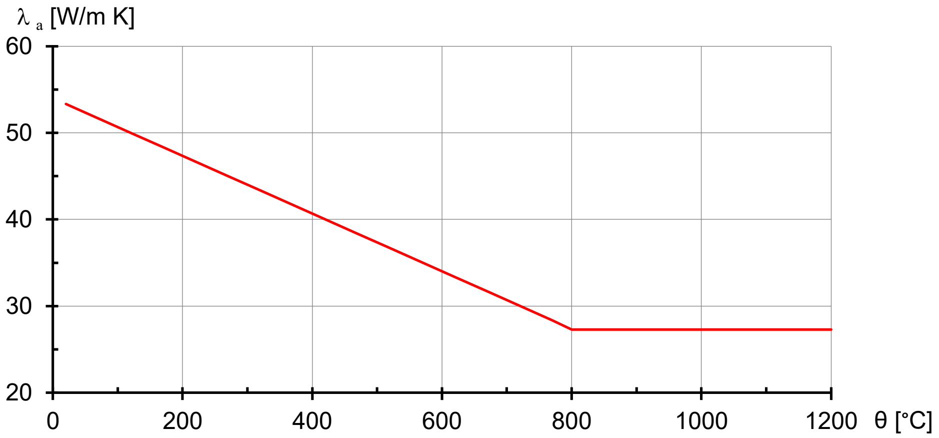
Temperature-dependent raw density ratio of concrete

Steel

All steel types are assigned the temperature-dependent *specific heat capacity* and the *thermal conductivity* as specified in EN 1993-1-2.



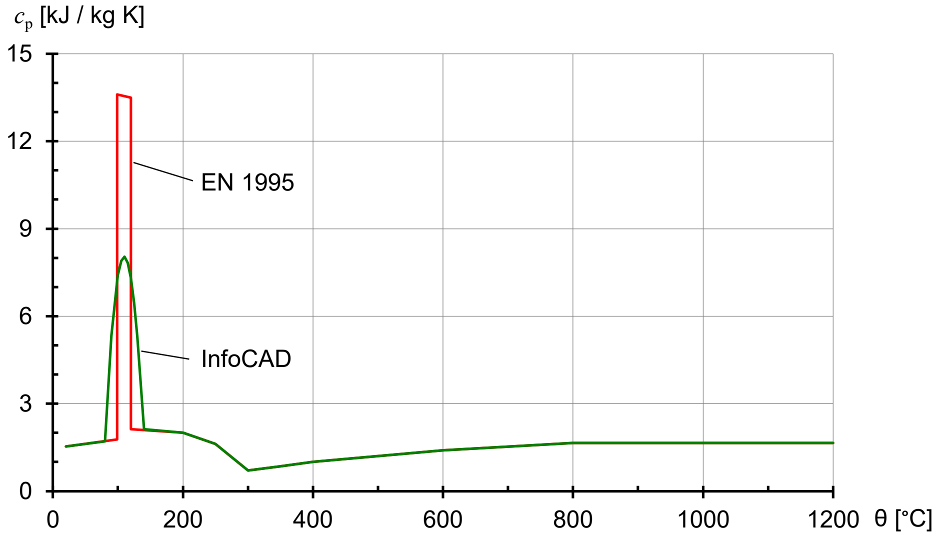
The temperature-dependent specific heat capacity for carbon steel



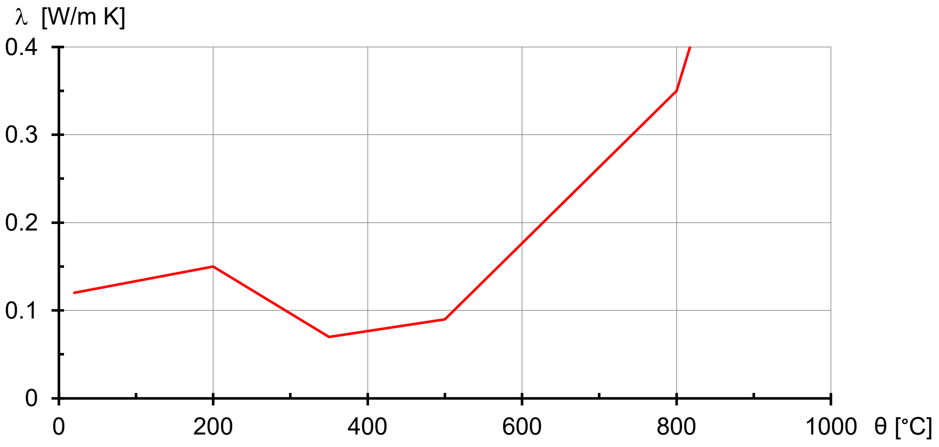
The temperature-dependent conductivity for carbon steel

Timber

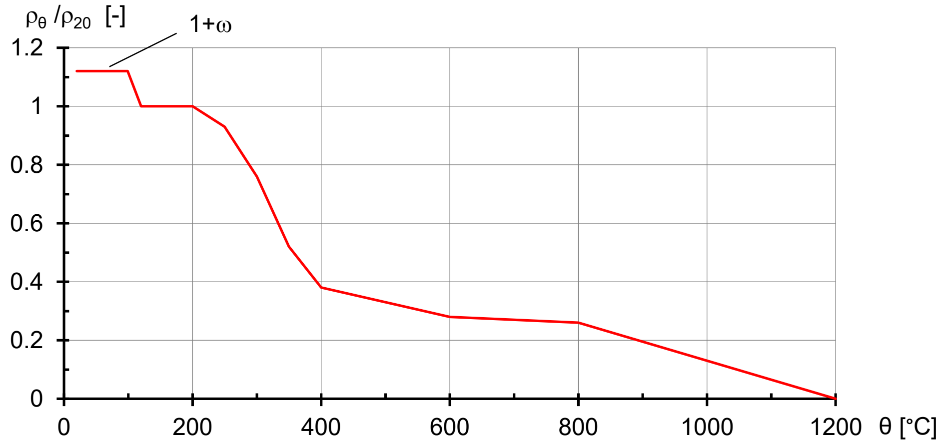
All timber types are assigned the temperature-dependent *specific thermal capacity*, the *thermal conductivity* and the *raw density* as specified in EN 1995-1-2.



The temperature-dependent specific heat capacity for timber



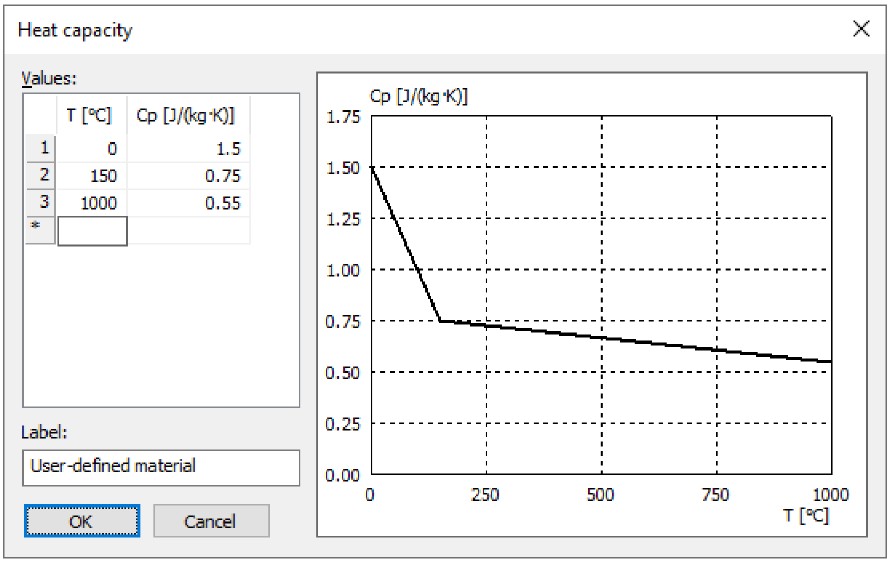
The temperature-dependent conductivity for timber



The temperature-dependent raw density ratio for timber with an initial moisture content ω of 12 %

User-defined material

User-defined section parts are used to specify, for example, insulating materials, where any temperature-dependent *specific heat capacity*, *apparent density* and *thermal conductivity* can be specified.



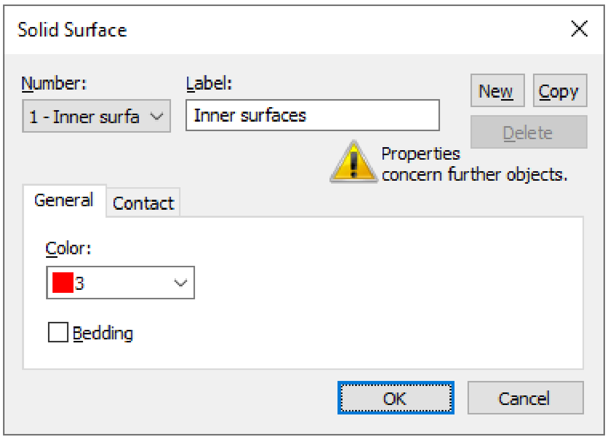
	T [°C]	Cp [J/(kg·K)]
1	0	1.5
2	150	0.75
3	1000	0.55
*		

Label: User-defined material

User-defined thermal conductivity

Solid Surface

With the context menu surface properties can be assigned to each surface of a solid model object. At contact faces the thermal conduction proceed in the same way as in a monolithic structure, if they have identical element dimensions and touch each other at the initial state.



Number: 1 - Inner surfa

Label: Inner surfaces

Properties concern further objects.

Color: 🟥3

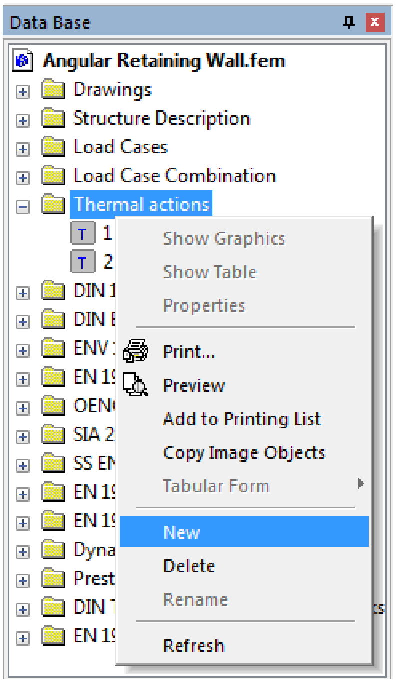
☐ Bedding

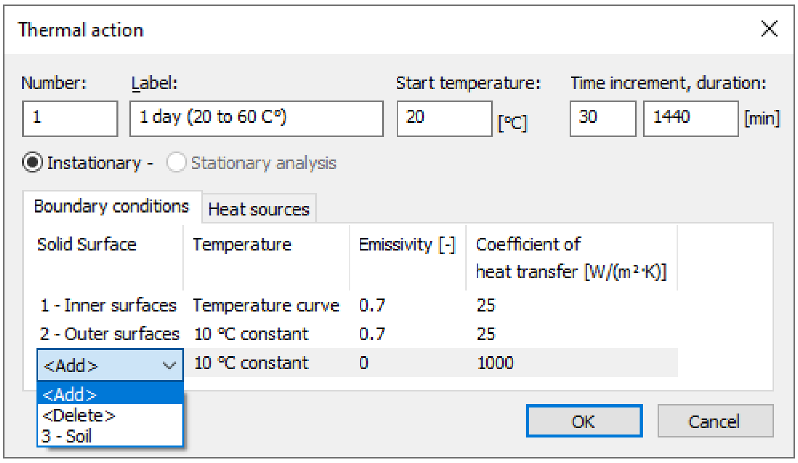
Thermal Actions

New thermal actions are created using the context menu of the data base. The analysis is performed separately for each thermal action.

In the dialog of the actual thermal action the following properties are defined:

- *Instationary* or *Stationary analysis*: For an instationary analysis additionally the start temperature [°C], the time increment and duration of the analysis [min] are to be specified.
- For each solid surface the boundary conditions described below can be assigned.
- Additionally a heat source can be defined for each section. For the power density [W/m³] of the heat source of the material a constant value or a user-defined time-dependent curve can be specified.





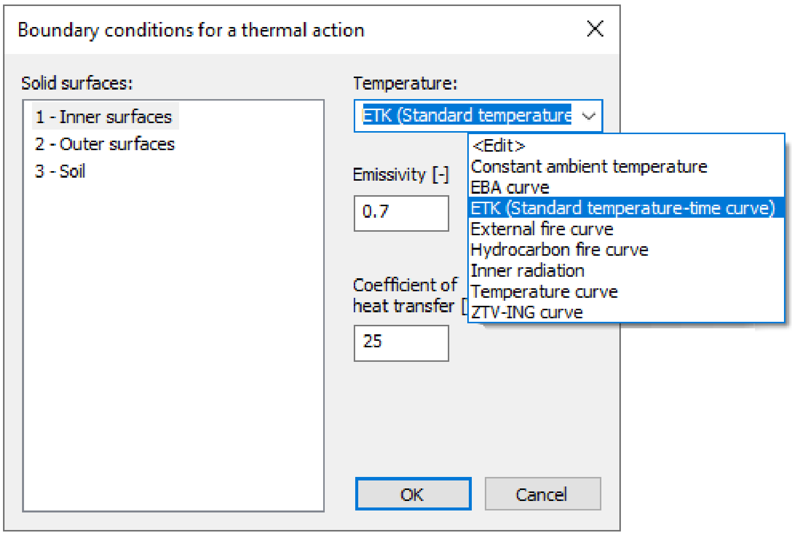
Number: 1
Label: 1 day (20 to 60 C°)
Start temperature: 20 [°C]
Time increment, duration: 30 1440 [min]

● Instationary - ○ Stationary analysis

Solid Surface	Temperature	Emissivity [-]	Coefficient of heat transfer [W/(m²·K)]
1 - Inner surfaces	Temperature curve	0.7	25
2 - Outer surfaces	10 °C constant	0.7	25
<Add>	10 °C constant	0	1000

<Add>
<Delete>
3 - Soil

Boundary conditions of solid surfaces



Solid surfaces:
1 - Inner surfaces
2 - Outer surfaces
3 - Soil

Temperature: ETK (Standard temperature

Emissivity [-]: 0.7

Coefficient of heat transfer [: 25

The thermal action on a surface is defined by the following boundary conditions:

- Temperature curve of the action, e.g. *ETK (Standard Temperature-Time Curve)*
- *Emissivity [-]*
- *Coefficient of heat transfer [W/(m² K)]*

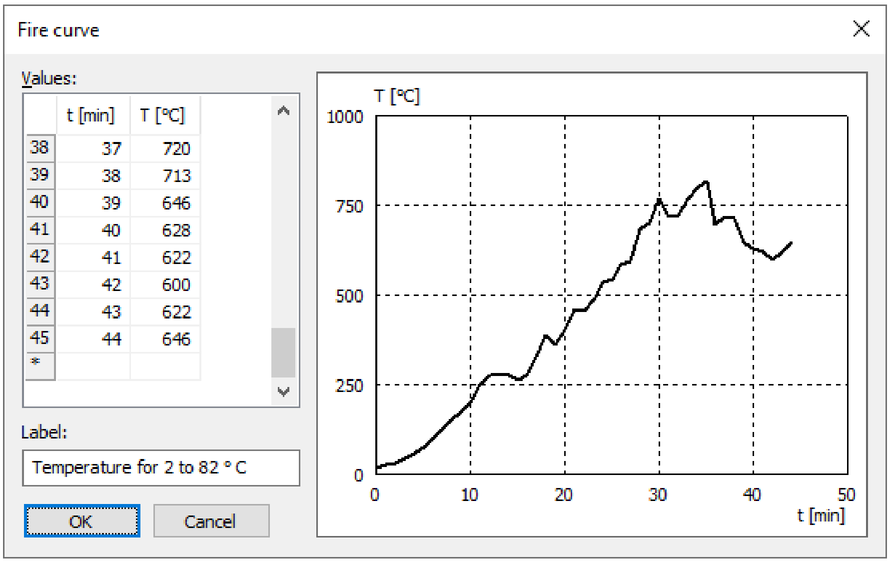
Surfaces which have no boundary conditions specified in this thermal action behave adiabatic. To surfaces which irradiate each other the property *inner radiation* can be assigned.

The convective coefficient of heat transfer can be taken from EN 1991-1-2, Appendix B.

The emissivity of concrete and construction steel surfaces should be determined according to EN 1992-1-2 or EN 1993-1-2, Chapter 2.2 (2) for a default value of 0.7. According to Appendix C of the standard, the emissivity for stainless steel should be set to 0.4.

The following temperature curves can be used to determine the progression of the action.

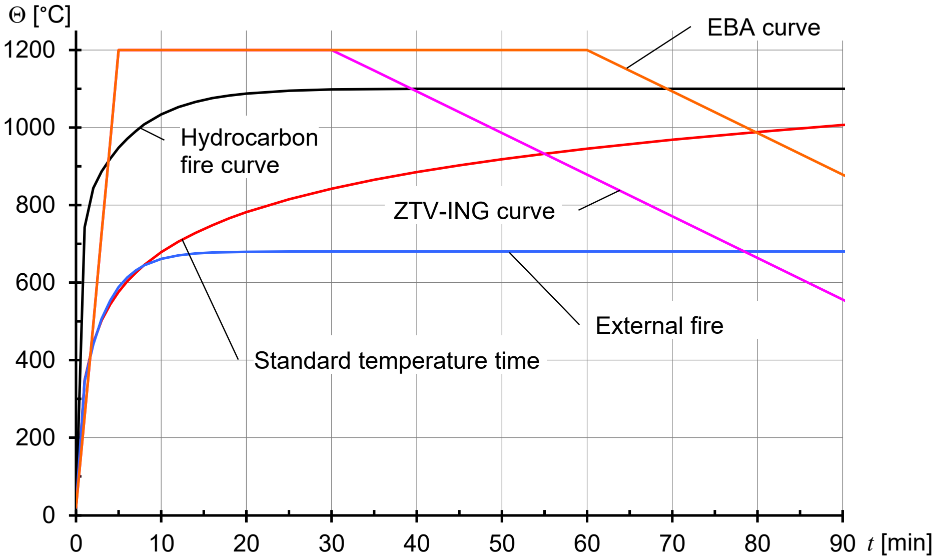
- *Constant ambient temperature*: The ambient temperature can be set.
- *EBA curve*: Fire curve according to EBA-Guideline.
- *ETK (Standard Temperature-Time Curve)*: Fire curve according to EN 1991-1-2 or prEN 13501-2.
- *External fire curve*: Fire curve according to EN 1991-1-2.
- *Hydrocarbon fire curve*: Fire curve according to EN 1991-1-2.
- *Inner radiation*: Surfaces which irradiate or are affected by radiation.
- *User-defined temperature curve*: Temperature curve defined by the user.
- *ZTV-ING curve*: Fire curve according to ZTV-ING Part 5.



	t [min]	T [°C]
38	37	720
39	38	713
40	39	646
41	40	628
42	41	622
43	42	600
44	43	622
45	44	646
*		

Label: Temperature for 2 to 82 ° C

User-defined temperature curve



Fire curves according to EN 1991-1-2, ZTV-ING Part 5 resp. EBA-Guideline

Thermal Analysis

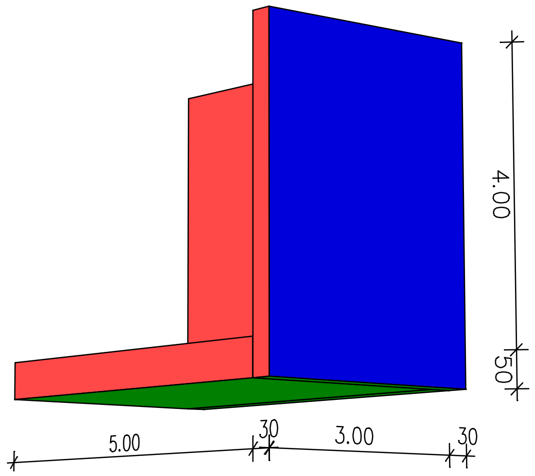
With the function *Thermal analysis* of the analysis menu all defined thermal actions are calculated.

Examples

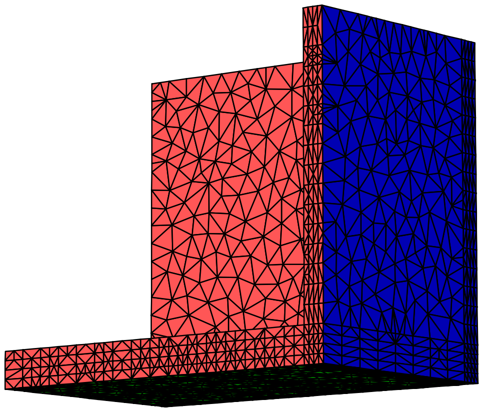
The following examples shall demonstrate possible applications of the program and also be used for validation of the attained results.

Instationary Temperature Progression in an Angular Retaining Wall

This example simulates the temperature progression during one day in an angular retaining wall.



Solid model



Tetrahedron elements

Material properties

	No.	Type	E-Modu. [MN/m²]	G-Modu. [MN/m²]	Poiss. ratio	alpha.t [1/K]	gamma [kN/m³]
1	1	C35/45-EN	34000	14200	0.20	1.000e-05	25.000

Thermal properties

	Material	Raw density [kg/m³]	Humidity ratio [%]	Heat capacity [J/(kg·K)]	Heat conductivity [W/(m·K)]
1	1 - C35/45-EN	2300	1.5		

Bedding

	No.	Start point [MN/m³]			End point [MN/m³]			Bedding width [m]		
		kbx	kby	kbz	kbx	kby	kbz	bx	by	bz
1	1	10	10	100						

The bedding performs in the direction of the axes of the local element or surface system.

Solid surfaces

	Number	Label	Color	Bed-ding	Slave of	Factor Contact-stiffness	Coefficient of friction	Influence radius [m]
1	1	Air side	🟥					
2	2	Outside	🟦					
3	3	Soil	🟩	X				

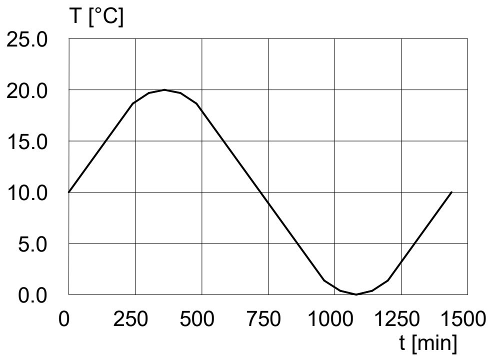
Thermal action 1 - 1 day (0 to 20 ° C)

Instationary analysis. Starting temperature = 10 °C; Time increment = 30 min; Duration = 1440 min

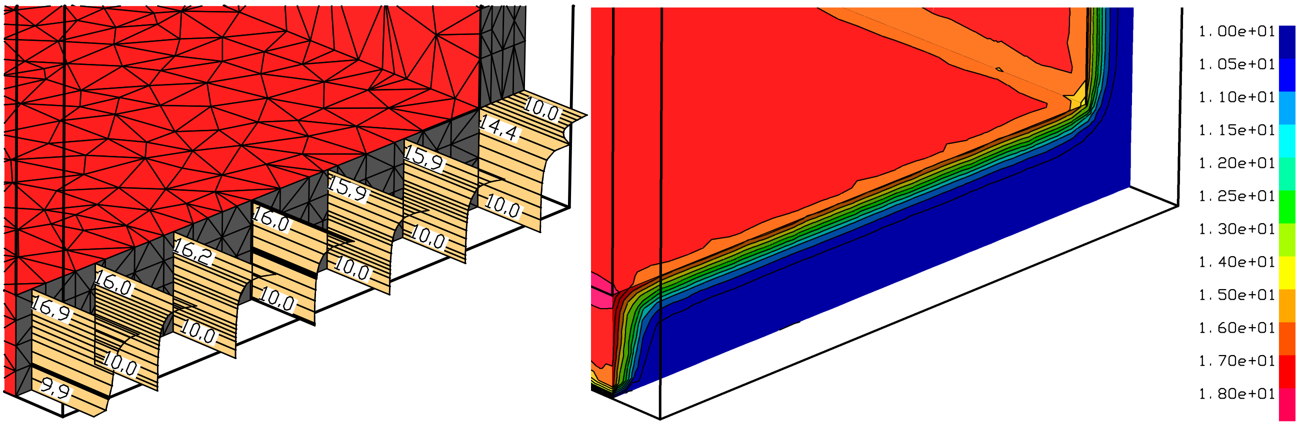
Boundary conditions

Solid Surface	Temperature	Emissivity [-]	Coefficient of heat transfer [W/(m²·K)]
1 - Air side	Temperature curve	0.7	25
2 - Outside	10 °C constant	0	1000
3 - Soil	10 °C constant	0	1000

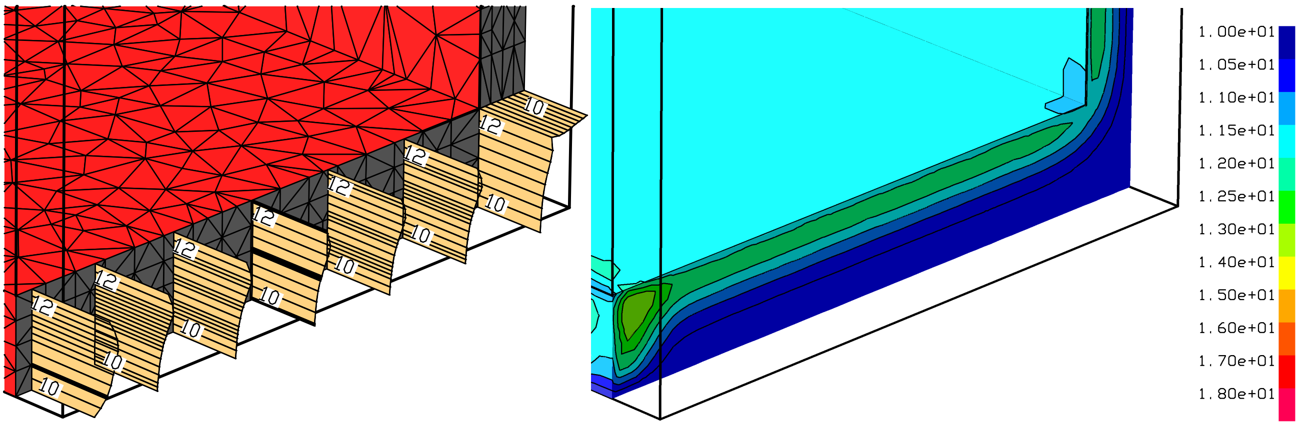
Temperature curve 1 - Days temperature 0 to 20 °C



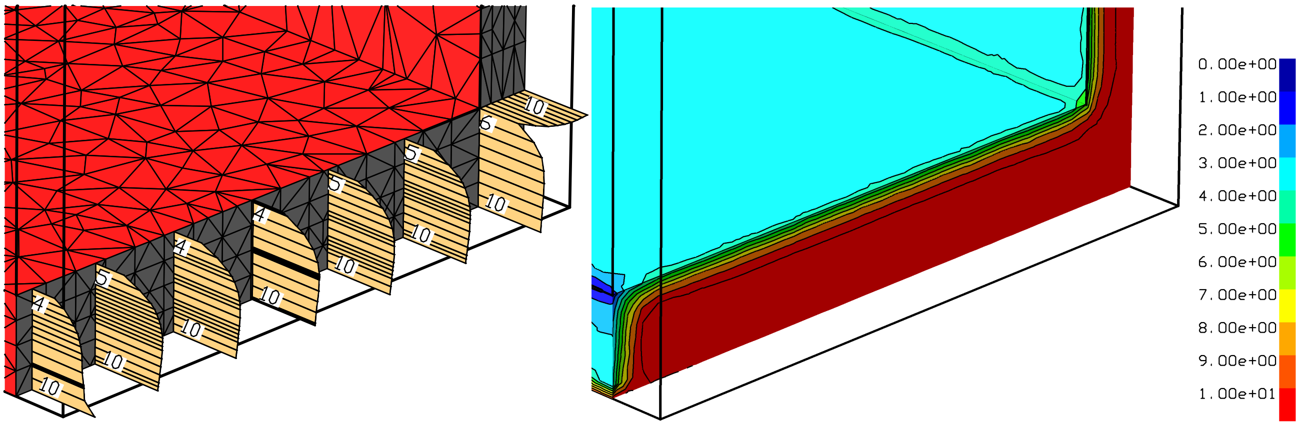
Pt.	t [min]	T [°C]
1	0	10
2	240	18.6603
3	300	19.6593
4	360	20
5	420	19.6593
6	480	18.6603
7	960	1.33975
8	1020	0.340742
9	1080	0
10	1140	0.340742
11	1200	1.33975
12	1440	10



Temperature distribution [°C] after 6 hours at the intersection face between wall and foundation slab

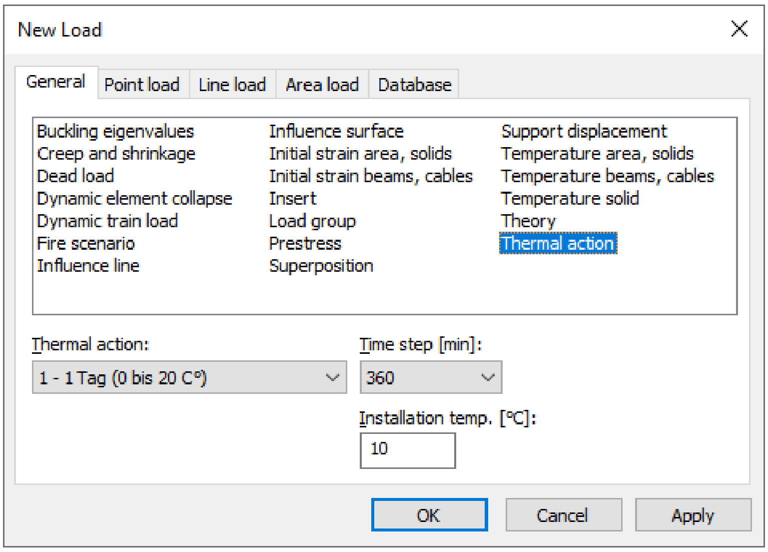


Temperature distribution [°C] after 12 hours at the intersection face between wall and foundation slab



Temperature distribution [°C] after 18 hours at the intersection face between wall and foundation slab

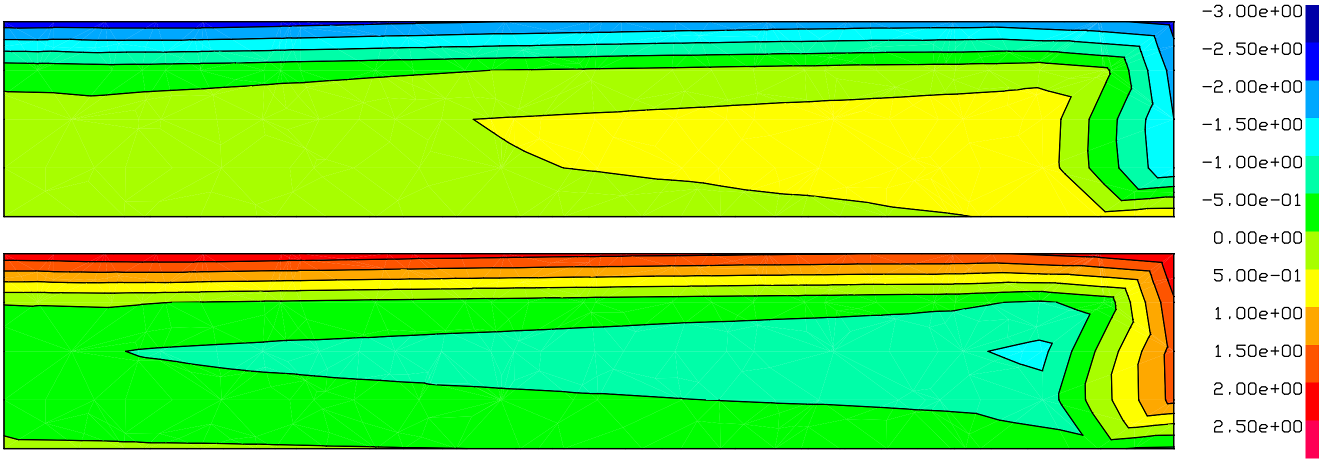
The stresses were determinated at the time of the maximum and minimum air temperature (6/18 h). A installation temperature of 10 °C was assumed.



Thermal action: 1 - 1 Tag (0 bis 20 C°)

Time step [min]: 360

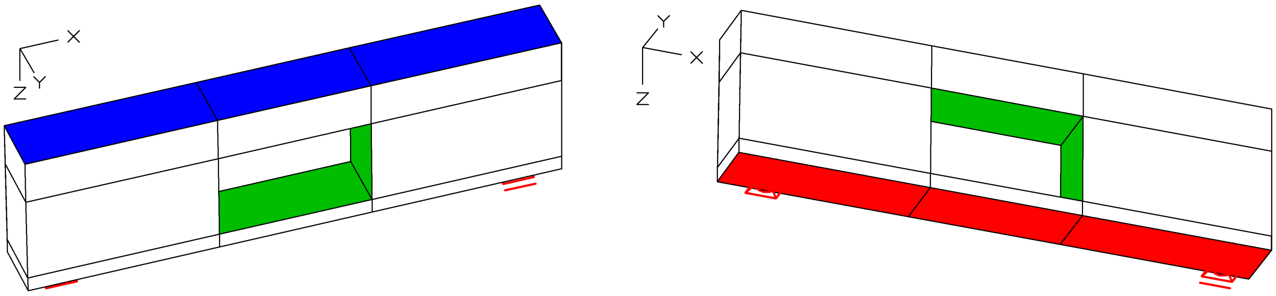
Installation temp. [°C]: 10



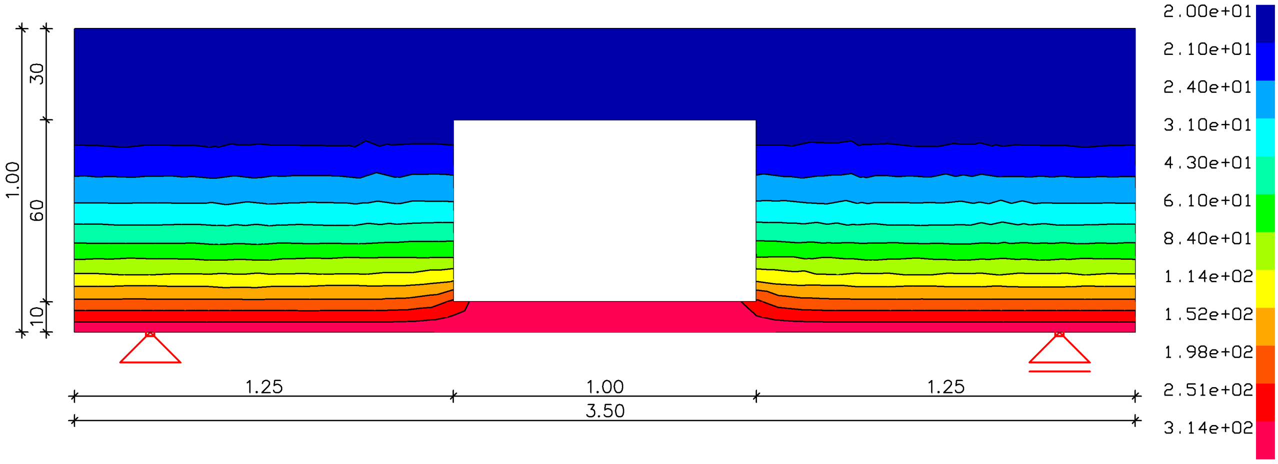
Stresses [MN/m²] after 6 and 18 hours perpendicular to the intersection face in the center of the foundation slab.

Comparative Calculation With and Without Radiation

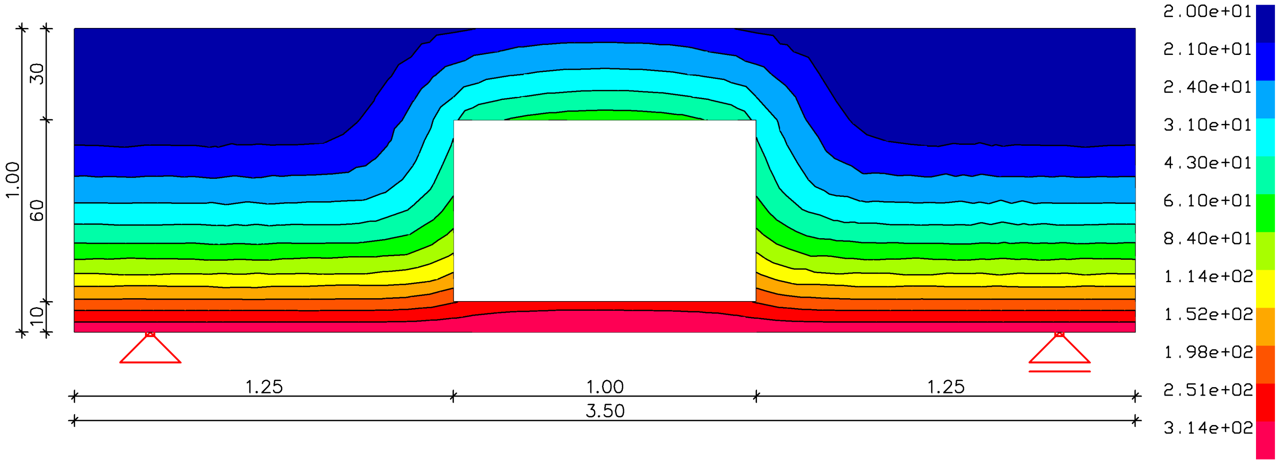
This example illustrates the effect of inner radiation on the temperature distribution. The pictured 40 cm thick concrete body C20/25 (EN 1992-1-1, raw density 2300 kg/m³, humidity ratio 2 %) has depending on the radiation considerably different temperature distributions after 10 hours under temperature exposure. The radiation is applied at the four inner surfaces of the aperture. All defined surfaces have an emissivity of 0.7. The coefficient of heat transfer is 25 W/(m²·K). Surfaces to which no boundary conditions were assigned (white) behave adiabatic.



Solid model with surfaces 1. constant temperature 400 °C (red); 2. radiation (green); 3. outer surface 20 °C (blue).



Temperature T [K] without inner radiation (adiabatic) after 10 h



Temperature T [K] with inner radiation after 10 h

Stationary Temperature Destribution

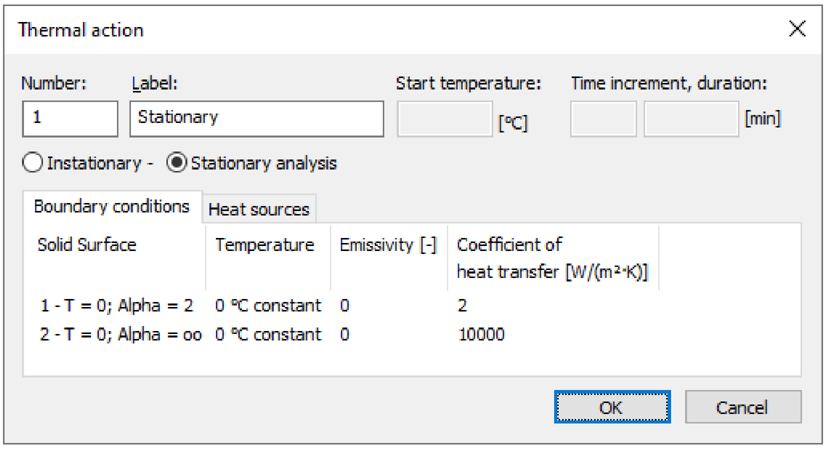
In the following example a stationary temperature distribution with heat source is calculated. The example was taken from the textbook *Methode der finiten Elemente*, Schwarz (1984), p. 285.

Task definition

- Heat source $\dot{q}$ = 20 W/m³ in G
- T = 0 °C at AB
- Adiabatic surface at BD, DE, EF, LM, MA
- Coefficient of heat transfer α = 2 W/(m²·K), T = 0 °C at FHIKL
- Conductivity 1.0 W/(m K)

Solid surfaces

	Number	Label	Color	Bed-ding	Slave of	Factor Contact-stiffness	Coefficient of friction	Influence radius [m]
1	1	T = 0; Alpha = 2	🟩					
2	2	T = 0; Alpha = oo	🟦					



Thermal action

Number: 1

Label: Stationary

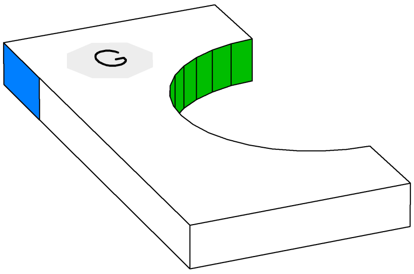
Start temperature: [°C]

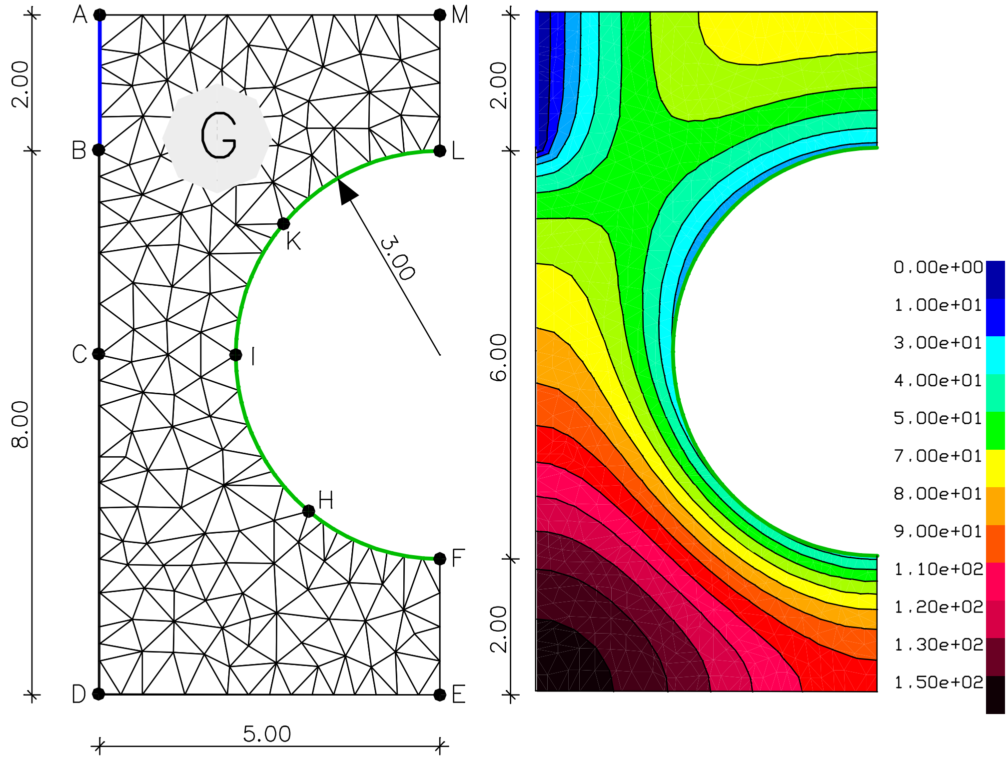
Time increment, duration: [min]

○ Instationary - ● Stationary analysis

Boundary conditions | Heat sources

Solid Surface	Temperature	Emissivity [-]	Coefficient of heat transfer [W/(m²·K)]
1 - T = 0; Alpha = 2	0 °C constant	0	2
2 - T = 0; Alpha = oo	0 °C constant	0	10000





Element mesh, dimensions [m] and temperature distribution [K]

References

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